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ON THE GAS FLOW RATE THROUGH A MACH DISK IN AN UNDEREXPANDED JET

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An air jet with an excess static pressure at a regime of escape into quiescent air is considered. The range of Mach number variation is 1–3, and that of the inefficiency ratio 1–1000. An analysis has been made on the basis of the well-known experimental-calculated approximation dependences for stationary expanding flows. We solve the inverse problem where the parameters at the nozzle exit and the geometric characteristics of the initial section of the jet are known and it is required to find the remaining quantities corresponding to them. In the present analysis, this is the rate of flow through a central discontinuity and the integral characteristic of the magnitude of the total pressure loss.

Keywords: *stationary jet, gas flow rate, Mach disk, total pressure losses, heat flux.*

Introduction. The gasdynamic structure of the entry section of an underexpanded axisymmetric jet escaping into a flooded space is the subject of a multitude of both theoretical and experimental investigations [1–5]. Here, it is convenient to represent the governing parameters as the dynamic ones: the inefficiency ratio n and the flow Mach number M_a; thermophysical: adiabatic index k and the ratio of the total temperatures of the gases of the jet and of the environment T^*/T_0; geometrical: the cone angle of the nozzle ϑ. If the regime in the mixing layer of the jet is laminar, then the governing parameters also include the Reynolds number.

The main effects in an underexpanded jet are predominantly attributable to the gasdynamic parameters. The influence of the temperature factor on the jet characteristics shows up only in the regimes that correspond to the transition to a rarefied flow. The contribution of the adiabatic index on the distribution of the Mach numbers along the symmetry axis and in the flow is significant, but the linear scale of the entry section of the jet is $\sim\sqrt{k}$. The cone angle of the nozzle begins to exert its influence on the flow structure only at a full apex angle larger than 40^oC. Thus, the basic governing parameters are the flow Mach number and the ratio of the static pressure in the jet to the ambient pressure.

According to the experimental data of [6], the role of the influence of the inefficiency ratio on the structure of the first "barrel" of a stationary sonic turbulent underexpanded jet escaping into a flooded space is shown in Fig. 1. The Mach disk and the reflected shock are depicted conventionally. The linear dimensions are related to the exit section diameter of the nozzle d.

The change in the distance to the Mach disk, its diameter, and the maximum diameter of the hanging shock are equal to about $\sqrt{n}$. An approximate estimation of the flow turning angle δ in a small vicinity of the nozzle edge (within the framework of the model of an ideal fluid) can be obtained by considering the Prandtl–Meyer plane flow.

Statement of the Problem. In an underexpanded supersonic jet the greatest attention is directed to the central shock or the Mach disk — in a free jet this is a curvilinear surface with a downstream concavity that intersects the axis at a right angle. The explanation is that it is easily identified visually and, from the practical point of view, it represents in essence a direct compression shock and is a source of the discontinuity of a whole number of kinematic and thermodynamic quantities. The static pressure and temperature increase, the total pressure drops, the entropy increases, and the flow acquires a subsonic velocity. Only the total enthalpy and temperature are preserved.

Since the passage of the compression shock by the gas is not an isentropic process but is accompanied by a transition of the mechanical energy into the thermal one, knowledge of the change in the parameters is of great interest in this case. In engineering applications precisely integral characteristics such as the mass flow rate and the total

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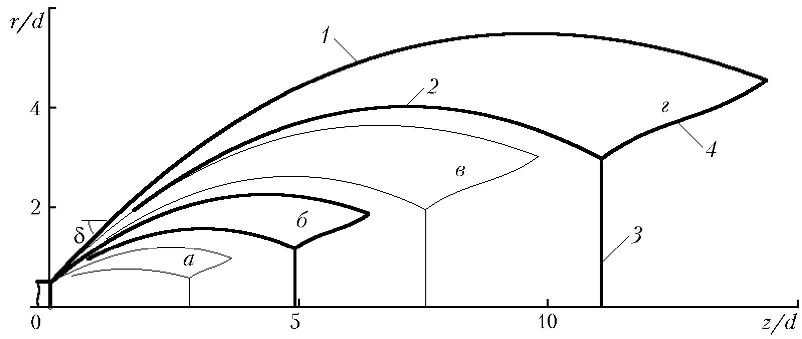

Fig. 1. Conventional scheme of the initial section of the underexpanded air jet at the flow Mach number M_a = 1, and different inefficiency ratios (a) n = 10; b) 30; c) 70; d) 150): 1) jet boundary; 2) hanging shock; 3) central shock (Mach disk); 4) reflected shock.

pressure loss are primarily important. If a jet at distances comparable with the entry section length interacts with the elements of structures, the magnitude of the rate of mass flow through the region of the flow core determines both the thermal loading and dynamic effect.

As is seen from Fig. 1, with increase in the ratio of the static pressure in the jet to the ambient pressure, the hanging shock is located nearer and nearer to the jet boundary, whereas the region of flow between the shock and the boundary that is subject to the influence of ambient conditions is more and more compressed. An analogous picture, but only with account for the increment of the axial direction, is also observed at high Mach numbers. Thus, in a supersonic jet the flow in the vicinity of the symmetry axis is independent of the conditions on the boundary and with a sufficient enough accuracy can be considered as in the case of escape of an inviscid jet into vacuum.

Under the conditions of the formulated assumptions at the nozzle for the given parameters at the nozzle edge, knowing the geometric characteristics of the jet we can determine the rate of gas flow through the Mach disk and the value of total pressure losses.

Method of Calculation. For the given M_a and n the axial coordinate and radius of the central shock are determined. According to the data of [6], the distance to the Mach disk and its radius are respectively determined by

$$z/d = (0.8 + 0.085\,(M_a - 2.1)^2)\, M_a \sqrt{n - 0.5} \quad (M_a = 1 - 3.6)\,,$$

$$r/d = 0.325\,(\sqrt{n} - 1)\cos[0.683\,(M_a - 1.9)] \quad (M_a = 1 - 4.2)\,.$$

In order to allow for the change in the flow parameters in the transverse direction, the radius should be split into circular segments with a certain step the number of which is designated by m.

The flow in the section between the nozzle edge and the Mach disk is isentropic; therefore, knowing the parameters of the escape at the edge and any parameter directly before the shock, it is possible, with the aid of gasdynamic functions, to find also the remaining quantities at this point. As such a parameter it is convenient to select the Mach number, which is the basic self-similarity number in high-velocity gas flows. As a result, the problem on determination of the gas flow rate is reduced to finding coefficients.

By virtue of the flow isentropicity, for the change in the static temperature, flow velocity, and in its density, the following relations are valid:

$$\frac{T_i}{T_a} = \left(1 + \frac{k-1}{2} M_a^2\right) \Big/ \left(1 + \frac{k-1}{2} M_i^2\right), \quad \frac{v_i}{v_a} = \frac{M_i}{M_a}\sqrt{\frac{T_i}{T_a}}\,, \quad \frac{\rho_i}{\rho_a} = \left(\frac{T_i}{T_a}\right)^{\frac{1}{k-1}}.$$

The ratio between the areas of the current circular fragment of the central shock sections and of the nozzle exit is determined from obvious geometric relations.

If we multiply the coefficients for the change in the velocity (v_i/v_a), density (ρ_i/ρ_a), and area ratio (F_i/F_a) between themselves and sum up over the radius, we can easily see that the expression represents the ratio of the rate of gas mass flow through the Mach disk to the total gas flow rate in the jet. Then finally we can write

$$\frac{G}{G_a} = \sum_{i=1}^{m} \frac{F_i}{F_a} \frac{M_i}{M_a} \left(\frac{T_i}{T_a}\right)^{\frac{k+1}{2(k-1)}} \cos \alpha_i ,$$

where α_i is the angle of deviation of the absolute velocity v_i on the current radius from the axial direction.

Further, knowing the Mach number directly ahead of the central shock, it is possible, according to [7], to determine also the coefficient of total pressure recovery as in the case of gas passage through a direct shock in the one-dimensional consideration:

$$\chi_i = \left.\frac{P_2^*}{P_1^*}\right|_i = \left(\frac{2k}{k+1} \mathrm{M}_i^2 - \frac{k-1}{k+1}\right)^{-\frac{1}{k-1}} \left(\frac{\frac{k+1}{2} \mathrm{M}_i^2}{1 + \frac{k-1}{2} \mathrm{M}_i^2}\right)^{\frac{k}{k-1}} \qquad (i \in [1; m]) .$$

For ease of further consideration it is convenient here to pass from the total pressure recovery coefficient to that pressure component that is directly spent on heat formation. The area-averaged integral characteristic of the magnitude of losses will be

$$1 - \frac{P_2^*}{P_1^*} = 1 - \frac{\sum_{i=1}^{m} \chi_i F_i}{F} .$$

Thus, all the relations given above are resolvable at the known Mach number in an inviscid field of free expansion of the jet.

To find the parameters in this region, in the gas dynamics of escape processes by the present time the methods of characteristics* and finite-difference methods have been well verified that allow one, using the initial values of the parameters at the nozzle edge, to find the gas parameters in the entire supersonic portion of the flow adjacent to the nozzle. The method of characteristics allows one, for example, to determine also the position of the boundary of the jet and of the hanging shock (but nothing more, since for the distance to the Mach disk an additional condition is needed).

The calculation of the flow field of a freely expanding gas escaping from an axisymmetric nozzle was made in [9]. The results are presented as dependences of Mach numbers and of the lines of equal flow rate in the field of the jet on the parameters determining the flow: the flow Mach number at the nozzle edge, adiabatic index, and half-angle of the nozzle.

Since the employed approximating data on the structure of the initial section of the jet are presented irrespective of the angles of the nozzles investigated in experiments (it is only reported that their half-angles did not exceed 20^o), for supersonic Mach numbers at the nozzle edge 2 and 3 the values of the half-angles were arbitrarily selected equal to 5 and 10^o, respectively. The values at the intermediate points after the digitization of data were determined by interpolation. As already noted, the influence of this parameter on the structure of the initial section of the supersonic underexpanded jet is insignificant.

Results and Discussion. The data of calculation by the above-given algorithm are presented in Fig. 2. It is seen from the figure that the substantial static pressure losses typical of the gas passage through a direct shock correspond to a very small fraction of the rate of mass flow through the Mach disk. For example, in a sonic jet at n = 30

*The method was suggested in 1900, but, not being used, was forgotten, to be later rediscovered in [8]. At the present time it belongs to the history of gas dynamics.

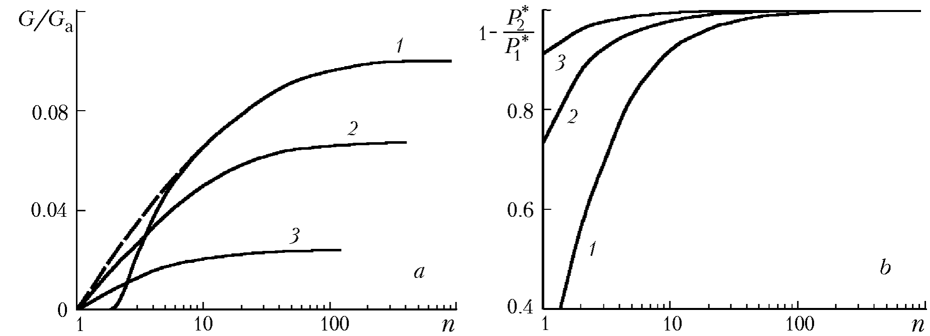

Fig. 2. Relative rate of mass flow through the Mach disk (a) and the total pressure loss coefficient (b): 1) M = 1; 2) 2; 3) 3.

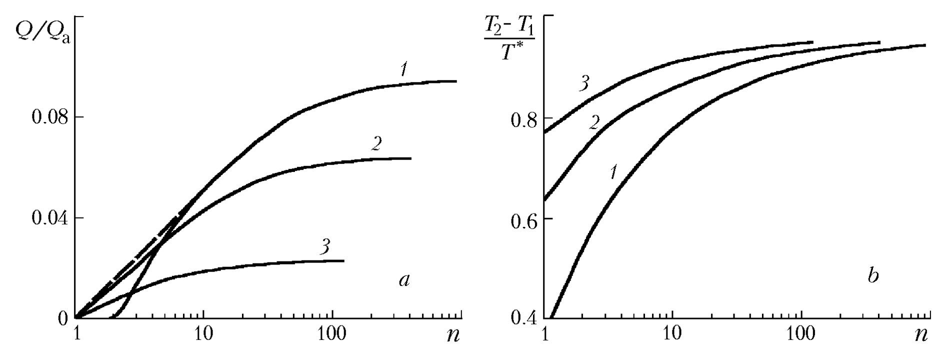

Fig. 3. Quantity of heat evolved due to the irreversibility of the process when the gas passes through the central shock (a) and the change in the static temperature (b): 1) M = 1; 2) 2; 3) 3.

(see Fig. 1) the coefficient of total pressure losses of 97.5% (irreversible losses) corresponds to the relative value of the mass flow rate equal only to 8.5%. There occurs a kind of mutual compensation of two factors, indicating the necessity of their joint consideration. It is the energy characteristic of the flow that may serve as such a criterion of their mutual influence.

The quantity of heat liberated due to the irreversibility of the process when the gas passes the Mach disk, the total heat content of the flow when leaving the nozzle, and their ratio will be written as

$$Q = Gc_p\,(T_2 - T_1)\,, \quad Q_a = G_a c_p T^*\,, \quad \frac{Q}{Q_a} = \frac{G}{G_a}\frac{T_2 - T_1}{T^*}\,.$$

The relative change in the temperature can be presented in the form [7]

$$\frac{T_2 - T_1}{T^*} = 1\Big/\left(1 + \frac{k-1}{2}\mathrm{M}_1^2\right)\frac{2\,(k-1)}{(k+1)^2\,\mathrm{M}_1^2}\,(\mathrm{M}_1^2 - 1)\,(1 + k\mathrm{M}_1^2)\,.$$

The graphical dependence allowing one to estimate the total pressure losses at a direct shock with account taken of the mass flow rate at this shock, with the comparison made in the units of the quantity of heat, is shown in Fig. 3.

Thus, the irreversible character of the total pressure losses leads to a rather insignificant redistribution of energy in the jet. In the example with M_a = 1 and n = 30 this is 7.3%. A comparison of the relations obtained allows one to consider the passage of the gas through the central shock from different standpoints and in their coordination.

The investigation carried out is the result of comparison of two works: an experimental and a computational one devoted to the same problem, that is, gas flow escape from an axisymmetric supersonic nozzle. But while in the former work a forming shock-wave structure in the initial section of a jet was investigated, the aim of the second

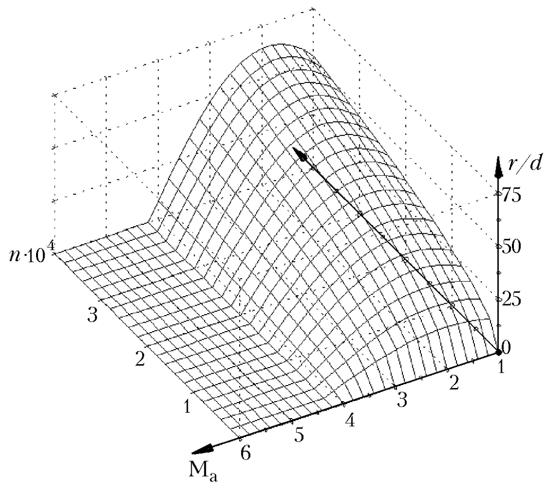

Fig. 4. Ratio between the radius of the central jump and the nozzle out diameter vs. the Mach number of the flow and inefficiency ratio.

work was to calculate the parameters of the field of a jet flow of an ideal perfect gas escaping into a vacuum. The highly understated values of the gas flow rate at small efficiency ratios and, correspondingly, of the heat flux for the sonic regime as compared to the supersonic one (see Figs. 2 and 3) is just the error of the assumption made — of the condition of gas expansion in a vacuum. This assumption is entirely valid for the far field of the jet, but the dependence of the distance to the central shock on the Mach number of the nozzle is linear (corrected values on the figures are shown by dashed lines).

The equation applied in the present analysis for the axial coordinate [6] is in close agreement with [2] in which the authors, as a result of processing experimental data on the distance to the central shock in underexpanded submerged jets of carbon dioxide, nitrogen, and helium, obtained the empirical dependence $z/d = 0.69\ \mathrm{M_a}\sqrt{kn}$ (n = 2–80; $\mathrm{M_a}$ = 1.5–3.3; k = 1.3, 1.4, 1.67). A better agreement is observed for the region of Mach numbers 1.5–2.5. These two experimental works logically supplement one another. The former points to the influence of the adiabatic index of the gas and the latter indicates the change of the parameter within a considerably wider range of initial data. The results of any of them can be extrapolated to the other.

Virtually the entire range of existence of the Mach disk in an underexpanded air jet is clearly reflected by Fig. 4. Here attention should be given to two moments: first, the maximum of the central shock diameter falls on $\mathrm{M_a}$ = 1.9 and is practically independent of the inefficiency ratio; second, the Mach disk does not exist at $\mathrm{M_a} > 4.2$. Thus, the reason for the decrease in the gas rate of flow through the central shock with an increase in the initial Mach number at the nozzle edge becomes evident.

The problem considered is directly related to a topic of great interest at the present time — the monitoring of the flow structure and its parameters. The thing is that, in practice, preceding the stationary escape of a gas, there is always a period of nonstationary outflow. But since fundamentally different statements are needed to observe two stages of the same process in an experiment, the nonstationary and stationary processes are the subjects of different investigations. The idea of integral consideration of the flow pattern at the initial stage of jet formation and development of a stationary regime was suggested in [10] and extensive data on the monitoring of the structure and parameters of an underexpanded behind a shock wave were summed up in [11]. The specific feature of the nonstationary stage consists in a greater expansion in the section adjacent to the nozzle exit and correspondingly larger total pressure losses at shocks and on sudden expansion. But something separate and specific can be understood much better when its place in the integral picture is well understood. Precisely the quantitative estimate of the total pressure losses in a stationary jet is that asymptote toward which the development of the process of initial stage of formation of the nonstationary flow structure will tend.

CONCLUSIONS

1. The highest rate of flow through the Mach disk in a highly underexpanded jet corresponds to the sonic regime of outflow, even though the maximum diameter of the central shock corresponds to $M_a \approx 2$.

2. For all Mach numbers of the regions of the existence of the central shock in an underexpanded jet the gas flow rate function of the nonefficiency ratio is asymptotic.

3. The irreversibility of the process of gas passage through the central shock in the entire range of flow Mach numbers and inefficiency ratios of the flow leads to a less than 10% redistribution of energy in the jet.

4. The trends revealed point to the possibility of monitoring the total pressure losses in an underexpanded supersonic jet by replacing the outflow regimes.

NOTATION

c_p, heat capacity at constant pressure, J/(kg·K); d, diameter of the nozzle exit section, m; F, area, m^2; G, flow rate, kg/sec; k, adiabatic index; m, number of circular fragments of the division of the central shock; M, flow Mach number; n, inefficiency ratio of a jet (ratio of the static pressure at the nozzle exit to environmental pressure); P, pressure, Pa; Q, heat flux, J/sec; r, radial coordinate, m; T, temperature, K; v, flow velocity, m/sec; z, axial coordinate, m; α, angle of streamline deviation from the axial direction, deg; δ, angle of turning of the flow during its escape from the nozzle, deg; ϑ, angle of nozzle conicity, deg; ρ, density, kg/m^3; χ, coefficient of total pressure recovery. Subscripts and superscripts: a, at the exit from the nozzle (in the cut plane); i, at the current radius for a circular fragment of the central shock; 0, in the environment; 1 and 2, directly ahead of the central shock and behind it; *, retarded flow.

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